

Change of coordinate

Equations (5) - (8) are to be solved on a domain $z \in [0, L]$ with a moving upper boundary at $z = L(t)$. For computational convenience, we introduce a new spatial coordinate $\zeta = z/L(t)$ in order to maintain a fixed domain $\zeta \in [0, 1]$. The change of coordinate implies the following change in partial derivatives:

$$\frac{\partial}{\partial z} \rightarrow \frac{1}{L} \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial t} \rightarrow -\zeta \frac{\dot{L}}{L} \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial t}$$

We also introduce the scaled velocity $u(\zeta) = v(z)/L$. In the new coordinates, equations (5) - (8) become

$$\begin{aligned} \frac{D_s}{L^2} \frac{d^2 s}{d\zeta^2} &= \beta(s) \\ \frac{\partial x}{\partial t} &= g \cdot P - \frac{\partial(xu)}{\partial \zeta} + \zeta \frac{\dot{L}}{L} \frac{\partial x}{\partial \zeta} - n_{GFP} \cdot x \\ \frac{du}{d\zeta} &= \mu(s) \\ \frac{dL}{dt} &= Lu(\zeta, t) \end{aligned}$$

The same change of coordinate is applied in solving equations (12) - (16) and (17) - (22).

References

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