

Text S1

1 Perturbative theory, second order expansion

The second-order coefficients of the general expansion read

$$x_2 = \frac{1}{6f_a^3 f_{xx}^3} (-f_{xa}^3 f_{xx} g^2 g_x^2 + f_a f_{xa}^2 g \\ g_x (-f_{xxx} g g_x + 6f_{xx} g_x g_{xx} + 2f_{xx} g g_{xxx}) + f_a (-f_{xa} f_{xx}^2 g^2 g_x^2 + \\ f_a (-f_a f_{xxx} (g_x^2 + g g_{xx})^2 + 2f_{xx}^2 g g_x (2g_x g_{xa} + g_a g_{xx} + g g_{xxa}) \\ - 2f_{xx} (g_x^2 + g g_{xx}) (f_{xx} g g_x - 3f_a g_x g_{xx} - f_a g g_{xxx})) + f_{xa} \\ (f_{aa} f_{xx}^2 g^2 g_x^2 + f_a (2f_a f_{xx}^2 g g_x (g_a g_x + g g_{xa}) + 2f_a f_{xxx} g g_x (g_x^2 + g g_{xx}) \\ + f_{xx} (2f_{xx} g^2 g_x^2 + f_a (g_x^4 - 4g g_x^2 g_{xx} \\ - 6g_x^3 g_{xx} + g^2 g_{xx}^2 - 6g g_x g_{xx}^2 - 2g(g_x(g+g_x) + g g_{xx}) g_{xxx}))))$$

$$a_2 = -\frac{1}{6f_a^3 f_{xx}} (-f_{xa}^2 g^2 g_x^2 + f_{aa} f_{xx} g^2 g_x^2 + 2f_a f_{xa} g g_x (3g_x g_{xx} + g g_{xxx}) + f_a^2 \\ (2f_{xx} g g_x (g_a g_x + g g_{xa}) + (g_x^2 + g g_{xx}) (g_x^2 + g g_{xx} - 6g_x g_{xx} - 2g g_{xxx})))$$

In the particular case of the bistable switch the above expressions lead to,

$$x_2 = -(3(1+x_0^2)^6 + 2a_0(1+x_0^2)^2(-12+x_0(7+3x_0(16+x_0(2+x_0(-12+5x_0))))) \\ + a_0^2(1+x_0(-24+x_0(2+3x_0(8+3x_0(6+24x_0-24x_0^3-7x_0^4+6x_0^6)))))) / \\ (12a_0^2 x_0(1+x_0^2)^2(-1+3x_0^2)^3)$$

$$a_2 = -\frac{1}{12a_0 x_0^4 (1+x_0^2)^3 (-1+3x_0^2)} ((1+x_0^2)^5 + 2a_0 x_0 (1+x_0^2)^2 \\ (2+x_0(12+x_0(1+3(-4+x_0)x_0))) \\ + a_0^2 x_0^2 (3+x_0(24+x_0(5+3x_0(16-3x_0(-1+x_0(8+x_0)))))))$$