**Text S3: Orthogonality of the Stat-POE model after transformation**

Except the variance component decomposition approach, the orthogonality of the Stat-POE models could also be checked by showing is a diagonal matrix in whichis the design matrix for the sample. As described in the original NOIA paper [15],

(C1)

needs to be satisfied, where

. (C2)

Given that  shown in equation (5) with , from (C1) and (C2) we derive the criteria for orthogonality when POE incorporated as

, (C3)

, (C4)

, (C5)

, (C6)

, (C7)

. (C8)

Except equation (C6), all of these criteria are satisfied by $S$ in equation (5). And for equation (C6), when  or  holds true.