## 1 Notation

| Symbol | Name | Context | Description |
| :---: | :---: | :---: | :---: |
| $x$ | Boolean Automaton | Automata | Binary-state automaton |
| $k$ | Number of inputs of $x$ | Automata | How many inputs determine the transitions of an automaton $x$ |
| F | Look-up table (LUT) of $x$ | Automata | The transition function of $x$ represented as a LUT ( $2^{k}$ entries). |
| $f_{\alpha}$ | LUT entry in $F$ | Automata | $k$-tuple combination of input states i.e. condition and corresponding transition |
| $c_{\alpha}$ | condition part in a LUT entry $f_{\alpha}$ | Automata | $k$-tuple combination of input states i.e. condition |
| $s_{\alpha}$ | transition in a LUT entry $f_{\alpha}$ | Automata | Boolean state prescribed as the transition in $f_{\alpha}$ |
| $\mathcal{B}$ | Boolean network | Networks | A graph of $N$ automata with directed edges (source node is input of end node) |
| X | set of automata in $\mathcal{B}$ | Networks | set of Boolean automata that constitute a BN $\mathcal{B}$ |
| $n$ | number of nodes in $\mathcal{B}$ | Networks | $n=\|X\|$ |
| $\boldsymbol{x}$ | network configuration | Networks | collection of the states of all nodes in a BN $\mathcal{B}$ |
| $X_{i}$ | set of inputs of node $x_{i}$ in $\mathcal{B}$ | Networks | Set of input nodes of $x_{i}$ |
| $k_{i}$ | in-degree of $x_{i}$ | Networks | Cardinality of $X_{i}$ |
| $F_{i}$ | Look-up table of $x_{i}$ | Networks | Transition function represented as a LUT |
| $f_{i: \alpha}$ | LUT entry in $F_{i}$ | Networks | Sub-indices $i$ and $\alpha$, separated by ' $:$ ', are used to specify node and entry. |
| $\mathcal{A}_{i}$ | An attractor of $\mathcal{B}$ | Networks | A specific (index $i$ ) fixed-point or periodic attractor of a BN $\mathcal{B}$. |
| $\sigma(\boldsymbol{x}) \rightsquigarrow \mathcal{A}$ | Dynamic trajectory of $\boldsymbol{x}$ to $\mathcal{A}$ | Networks | This notation is used to represent that the trajectory of some configuration $\boldsymbol{x}$ is known to converge to $\mathcal{A}$ |
| \# | Wildcard symbol | Wildcard | If this symbol appears in a condition, the variable it represents can be in any state. |
| $F^{\prime}$ | wildcard-schema redescription of $F$ | Wildcard | LUT where entries are wildcard schemata |
| $f_{v}^{\prime}$ | a wildcard schema in $F^{\prime}$ | Wildcard | An entry in $F^{\prime}$ is like an entry in $F$ but its condition part can have wildcard symbols. |
| $\Upsilon_{v}$ | Entries $f_{\alpha} \in F$ in $f_{v}^{\prime}$ | Wildcard | The set of original LUT entries in $F$ redescribed by a single wildcard schema $\Upsilon_{v} \equiv\left\{f_{\alpha}: f_{\alpha} \mapsto f_{v}^{\prime}\right\}$ |
| ${ }^{\circ}{ }_{m}$ | Position-free symbol | 2-Symbol | If a variable in the condition part of a schema is marked with this symbol, it can exchange places with any other variable in the same schema marked with the same symbol. Index $m$ used to distinguish subsets of identically-marked inputs |
| $\beta$ | Depth of search for two-symbol schemata | 2-Symbol | Defines the minimum number of wildcard schemata in a two-symbol redescription. |
| $F^{\prime \prime}$ | 2-symbol redescription of $F$ | 2-Symbol | LUT where entries are two-symbol schemata |
| $f_{\theta}^{\prime \prime}$ | a 2-symbol schema in $F^{\prime \prime}$ | 2-Symbol | An entry in $F^{\prime \prime}$ is like an entry in $F$ but its condition part can have wildcard and position-free symbols. |
| $\Theta_{\theta}$ | Entries $f_{\alpha} \in F: f_{\alpha} \longmapsto f_{\theta}^{\prime \prime}$ | 2-Symbol | The set of original LUT entries in $F$ redescribed by a single 2-symbol schema $\Theta_{\theta} \equiv\left\{f_{\alpha}: f_{\alpha} \mapsto f_{\theta}^{\prime \prime}\right\}$ |
| $\Theta_{\theta}^{\prime}$ | Schemata $f_{v}^{\prime} \in F^{\prime}: f_{v}^{\prime} \mapsto f_{\theta}^{\prime \prime}$ | 2-Symbol | The set of wildcard schemata in $F^{\prime}$ redescribed by a single 2-symbol schema $\Theta_{\theta}^{\prime} \equiv\left\{f_{v}^{\prime}: f_{v} \mapsto f_{\theta}^{\prime \prime}\right\}$ |
| $X_{\ell}$ | set of literal enputs in a schema $f^{\prime \prime}$ | 2-Symbol | The variables in the condition part of schema $f^{\prime \prime}$ that are specified in a Boolean state (not wildcard) |
| $\eta_{\ell}$ | size of literal-enput set in a schema $f^{\prime \prime}$ | 2-Symbol | $n_{\ell}=\left\|X_{\ell}\right\|$ |
| $X_{\ell}^{s}$ | state-s literal enputs in $f^{\prime \prime}$ | 2-Symbol | Subset $X_{\ell}^{s} \subset X_{\ell}$ of literal enputs in a specific state $s: s \in\{0,1\}$ |
| $X_{g}$ | group-invariant enput in a schema $f^{\prime \prime}$ | 2-Symbol | The set variables in the condition part of schema $f^{\prime \prime}$ that are marked with an identical position-free symbol, in every state they can take |
| $X_{g}^{s}$ | elements of $X_{g}$ in state $s$ | 2-Symbol | This notation is used to refer to the members of a group-invariant enput instantiated in a specific state $s$, that is $X_{g}^{s}=\left\{\forall x_{i} \in X_{g} \wedge x_{i}=s\right\}$ |
| $\eta$ | number of group-invariant enputs in $f^{\prime \prime}$ | 2-Symbol | Number of subsets of inputs marked with a distinct position-free symbol |
| $n_{g}$ | size of a single group-invariant enput $g$ in $f^{\prime \prime}$ | 2-Symbol | Number of inputs marked with the position-free symbol in $g$. |
| $n_{g}^{s}$ | a sub-constraint in $X_{g}$ on state $s \in\{0,1\}$ | 2-Symbol | specifies a group-invariant constraint in the set $X_{g}$, at least $n_{g}^{s}$ variables must be in state $s$ |


| Symbol | Name | Context | Description |
| :---: | :---: | :---: | :---: |
| $\tau$ | Threshold of a t-unit | Canalizing Maps | The firing activity threshold of a transition unit in the canalizing map of an automaton $x$ |
| $\underline{k}_{\mathrm{e}}(x)$ | lower-bound effective connectivity of $x$ | Automata control | Smallest number of inputs that, on average, determine the transition of $x$ when the states of all its inputs are equi-probable |
| $\bar{k}_{\mathrm{e}}(x)$ | upper-bound effective connectivity of $x$ | Automata control | Maximum number of inputs that, on average, are needed to determine the transition of $x$ when the states of all its inputs are equi-probable |
| $k_{\mathrm{r}}(x)$ | input redundancy in $x$ | Automata control | $k_{\mathrm{r}}(x)=k(x)-k_{\mathrm{e}}(x)$ |
| $\underline{k}_{\mathbf{s}}(x)$ | lower-bound input symmetry of $x$ | Automata control | Smallest number of inputs with with, on average, an input can 'switch places with' to determine the transition of $x$ when the states of all its inputs are equi-probable |
| $\bar{k}_{\mathrm{s}}(x)$ | upper-bound input symmetry of $x$ | Automata control | Largest number of inputs with with, on average, an input can 'switch places with' to determine the transition of $x$ when the states of all its inputs are equi-probable |
| $\hat{\boldsymbol{x}}$ | partial configuration | dynamic unfolding | A configuration of the BN where a subset of nodes is specified (in a Boolean state) while other nodes are unknown |
| $\sigma(\hat{\boldsymbol{x}}) \rightsquigarrow \mathcal{P}$ | dynamics from $\hat{\boldsymbol{x}}$ leads to outcome $\mathcal{P}$ | dynamic unfolding | Dynamic trajectory from $\hat{\boldsymbol{x}}$ ends in outcome pattern $\mathcal{P}$ (which can be a full attractor, or a partially specified steady-state configuration). |
| $\mathcal{P}$ | target outcome | Minimal Configs. | A target pattern comprises any configurations that share some property of interest. It can contain a single attractor, or a set of outcome patterns. |
| $\boldsymbol{x}^{\prime}$ | minimal configuration | Minimal Configs. | A configuration of the BN where a subset of nodes is specified (in a Boolean state) while other nodes are unknown such that $\sigma\left(\boldsymbol{x}^{\prime}\right) \rightsquigarrow \mathcal{P}$ |
| $\left\\|\boldsymbol{X}^{\prime}\right\\|$ or $\left\\|\boldsymbol{X}^{\prime \prime}\right\\|$ | Number of configurations $\boldsymbol{X}$ redescribed by a set of MCs | Minimal Configs. | The cardinality $\|\boldsymbol{X}\|$ of the set of configurations redescribed by a set of MCs may be counted exactly, or sampled. |
| $\sigma\left(\boldsymbol{x}^{\prime}\right) \rightsquigarrow \mathcal{P}$ | input-output relationship between a MC $\boldsymbol{x}^{\prime}$ and the target pattern it unfolds to, $\mathcal{P}$ | Minimal Configs. | Dynamic trajectory of a minimal configuration $\boldsymbol{x}^{\prime}$ ends in target pattern $\mathcal{P}$ (this can be a single (full or partial) configuration, or a set of these that defines a pattern). |

