Appendix S2 Estimation equations for parameters from fastBayesA

At convergence of the EM algorithm when $\widehat{\boldsymbol{\gamma}}^{(k-1)} \approx \widehat{\boldsymbol{\gamma}}^{(k)}$, the fastBayesA estimates of SNP effects ($\widehat{\boldsymbol{\gamma}}$) and fixed effects ($\widehat{\boldsymbol{\beta}}$) satisfy

$$\widehat{\boldsymbol{\gamma}} = \left[\mathbf{Z}'\mathbf{Z} + \widehat{\mathbf{D}}^{-1}\sigma_e^2\right]^{-1}\mathbf{Z}'\left(\boldsymbol{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}\right),$$

and

$$\widehat{\boldsymbol{\beta}} = \left(\mathbf{X}'\widehat{\mathbf{V}}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\widehat{\mathbf{V}}^{-1}\boldsymbol{y},$$

in which

$$\widehat{\mathbf{D}} = \mathrm{diag}\left\{\frac{\widehat{\gamma}_j^2 + \nu_{\boldsymbol{\gamma}} S_{\boldsymbol{\gamma}}^2}{\nu_{\boldsymbol{\gamma}} + 1}\right\}_{j=1}^{m},$$

and

$$\widehat{\mathbf{V}} = \mathbf{Z}\widehat{\mathbf{D}}\mathbf{Z}' + \mathbf{I}\sigma_e^2.$$

In this study the residual variance is assumed known from simulation. In most cases where the residual variance is unknown, the estimate in the kth step of EM iteration is calculated as

$$\{\widehat{\sigma}_e^2\}^{(k)} = \frac{\left[\boldsymbol{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}^{(k)} - \mathbf{Z}\widehat{\boldsymbol{\gamma}}^{(k)}\right]'\left[\boldsymbol{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}^{(k)} - \mathbf{Z}\widehat{\boldsymbol{\gamma}}^{(k)}\right]}{n - \mathrm{rank}(\mathbf{X})}.$$